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SOME PROPERTIES AND THEOREM ON FUZZY SUB-TRIDENT DISTANCE

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ABSTRACT

This paper introduces some simple properties and theorem based on Fuzzy Sub-Trident Distance along with the help of Trapezoidal Fuzzy Numbers. The results are discussed with suitable numerical example.

INTRODUCTION

Fuzzy Set Theory is introduced by Lotfi.A.Zadeh in the year 1965 [1].Later Liem Tran and Lucien Duckstein gave the Comparison of fuzzy numbers using a fuzzy distance measure in the year 2002 [3]. Later Shanhuo Chen and Chienchung Wang introduced the Fuzzy Distance of Trapezoidal Fuzzy Numbers in the year 2008 [4]. In the year 2012, A.Nagoorgani [5] gave a new operation on Triangular Fuzzy number for solving Fuzzy Linear Programming Problem. A New Method for Rank, Mode, Divergence and spread on Generalized Exponential Trapezoidal Fuzzy Numbers is given by Salim Rezvani in the year 2012 [6]. Arithmetic Operations on Generalized Trapezoidal Fuzzy Number and its Applications is given by Sanhita Banerjee and Tapan Kumar Roy in the year 2012[7]. In the year 2014, Pardhasaradhi and Ravi Shankar gave an idea on Fuzzy Distance Measure [8]. In this Paper, Some simple properties and theorem based on Fuzzy Sub-Trident Distance along with the help of Trapezoidal Fuzzy Numbers are given. This Paper consists of five sections. The preliminaries in the first section, Defining Trapezoidal, Positive Trapezoidal, Negative Trapezoidal Fuzzy Numbers in the second section, Fuzzy Sub-Trident Distance in the third section, Properties and Theorem based on Fuzzy Sub-Trident Distance in the fourth section and finally, the results are discussed with suitable numerical examples.

PRELIMINARIES

Definition 1. The Characteristic function $\mu_{\tilde{A}}$ of a crisp set $\tilde{A} \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e.,

$\mu_{\tilde{A}} : X \rightarrow [0,1].$ The assigned value indicates the membership grade of the element in the set $\tilde{A}$. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set. [2]

Definition 2. A fuzzy set $\tilde{A}$, defined on the universal set of $\Re$, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is   convex   i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2 \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$
$\forall x_1, x_2 \in \Re, \forall \lambda \in [0,1]$.

(ii) $\tilde{A}$ is normal i.e., $\exists x \in \Re$ such that $\mu_{\tilde{A}}(x) = 1$.

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.[2]

Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number $\tilde{A}$ is described at any fuzzy subset of the real line $\Re$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

- $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\Re$ to [0,1]

- $\mu_{\tilde{A}}(x) = 0, -\infty \leq x \leq c$

- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on [c,a]

- $\mu_{\tilde{A}}(x) = w, a \leq x \leq b$

- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on [b,d]

- $\mu_{\tilde{A}}(x) = 0, d \leq x \leq \infty$ where $0 \prec w \leq 1$ and a, b, c and d real numbers.

We denote this type of generalized fuzzy numbers as $\tilde{A} = (c, a, b, d; w)_{LR}$. When w=1, this type of generalized fuzzy number $\tilde{A} = (c, a, b, d)_{LR}$. When L(x) and R(x) are straight line, then $\tilde{A}$ is Trapezoidal fuzzy number and it is denoted by (c,a, b,d) [6].

Positive Trapezoidal Fuzzy Number:

A Positive Trapezoidal Fuzzy Number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where all $a_i's > 0 \forall i = 1,2,3,4$.

Negative Trapezoidal Fuzzy Number:

A Negative Trapezoidal Fuzzy Number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where all $a_i's < 0 \forall i = 1,2,3,4$.

Fuzzy Sub-Trident Distance
The distance between the two fuzzy numbers are calculated by using the new technique called the Fuzzy Sub-Trident Distance as follows:

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then   the   Fuzzy   Sub-Trident   Distance   is   given   by

$$FS_{Tri}dis\tan ce(\tilde{A}, \tilde{B}) = \left|\left\{\frac{1}{3}[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3]^{1/3}\right\}\right|.$$

Properties and Theorem on Fuzzy Sub-Trident Distance:
The following are the properties based on Fuzzy Sub-Trident Distance:

Property 1:

Let $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ be Trapezoidal Fuzzy Numbers. The Fuzzy Sub-Trident Distance of $\tilde{A}$ and $\tilde{B}$ is given by $FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B})$ and the Fuzzy Sub-Trident Distance of $\tilde{C}$ and $\tilde{D}$ is given by $FS_{Tri} dis \tan ce(\tilde{C}, \tilde{D})$ then

$FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B}) \leq FS_{Tri} dis \tan ce(\tilde{C}, \tilde{D})$ if $\tilde{A} \leq \tilde{C}$ and $\tilde{B} \leq \tilde{D}$.

Property 2:

If the Trapezoidal Fuzzy Numbers are Positive, then $FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B})$ is Positive.

Property 3:

If the Trapezoidal Fuzzy Numbers are Negative, then $FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B})$ is Positive.

The Theorem based on Fuzzy Sub-Trident Distance is as follows:

Theorem: 1

The Fuzzy Sub-Trident Distance $FS_{Tri} dis(\tilde{A}, \tilde{B})$ where $\tilde{A}, \tilde{B}$ are Trapezoidal Fuzzy Number then the following conditions hold:

$(i) FS_{Tri} dis(\tilde{A}, \tilde{B}) \geq 0, \, for \, \tilde{A}, \tilde{B} > 0.$

$(ii) FS_{Tri} dis(\tilde{A}, \tilde{B}) = FS_{Tri} dis(\tilde{B}, \tilde{A}).$

$(iii) FS_{Tri} dis(\tilde{A}, \tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}.$

$(iv) FS_{Tri} dis(\tilde{A}, \tilde{C}) \leq FS_{Tri} dis(\tilde{A}, \tilde{B}) + FS_{Tri} dis(\tilde{B}, \tilde{C}), where \, \tilde{A}, \tilde{B}, \tilde{C} \, are Trapezoidal Fuzzy Numbers.$

Proof:

$(i) \, To \, Prove \, FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B}) \geq 0.$

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ are Trapezoidal Fuzzy Numbers.

The proof is obvious from the definition of Fuzzy Sub-Trident Distance is given by

Thus for all values of $\tilde{A}, \tilde{B}, FS_{Tri} dis \tan ce(\tilde{A}, \tilde{B}) \geq 0.$

Hence the Proof.

$(ii) \, To \, Prove \, FS_{Tri} dis(\tilde{A}, \tilde{B}) = FS_{Tri} dis(\tilde{B}, \tilde{A}).$

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ are Trapezoidal Fuzzy Numbers.

$$FS_{Tri}dis\tan ce(\tilde{A}, \tilde{B}) = \left| \left\{ \frac{1}{3}\left[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 \right]^{\frac{1}{3}} \right\} \right|.$$

$$= \left| \left\{ \frac{1}{3}\left[(b_1 - a_1)^3 + (b_2 - a_2)^3 + (b_3 - a_3)^3 \right]^{\frac{1}{3}} \right\} \right|.$$

$$= FS_{Tri}dis\tan ce(\tilde{B}, \tilde{A}).$$

Thus $FS_{Tri}dis(\tilde{A}, \tilde{B}) = FS_{Tri}dis(\tilde{B}, \tilde{A})$.

Hence the Proof.

(iii) *To Prove* $FS_{Tri}dis(\tilde{A}, \tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}$.

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ are Trapezoidal Fuzzy Numbers.

$$FS_{Tri}dis\tan ce(\tilde{A}, \tilde{B}) = 0$$

$$\Leftrightarrow \left| \left\{ \frac{1}{3}\left[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 \right]^{\frac{1}{3}} \right\} \right| = 0$$

$$\Leftrightarrow (a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 = 0$$

$$\Leftrightarrow (a_1 - b_1)^3 = 0, (a_2 - b_2)^3 = 0, (a_3 - b_3)^3 = 0$$

$$\Leftrightarrow a_1 - b_1 = 0, a_2 - b_2 = 0, a_3 - b_3 = 0.$$

$$\Leftrightarrow a_1 = b_1, a_2 = b_2, a_3 = b_3.$$

$$\Leftrightarrow \tilde{A} = \tilde{B}$$

Thus $FS_{Tri}dis(\tilde{A}, \tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}$.

Hence the Proof.

(iv) *To Prove* $FS_{Tri}dis(\tilde{A}, \tilde{C}) \leq FS_{Tri}dis(\tilde{A}, \tilde{B}) + FS_{Tri}dis(\tilde{B}, \tilde{C})$.

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4), \tilde{C} = (c_1, c_2, c_3, c_4)$ are Trapezoidal Fuzzy Numbers.

$$FS_{Tri}dis\tan ce(\tilde{A}, \tilde{C}) = \left| \left\{ \frac{1}{3}\left[(a_1 - c_1)^3 + (a_2 - c_2)^3 + (a_3 - c_3)^3 \right]^{\frac{1}{3}} \right\} \right|.$$

$$\leq \left| \left\{ \frac{1}{3}\left[(a_1 - b_1 + b_1 - c_1)^3 + (a_2 - b_2 + b_2 - c_2)^3 + (a_3 - b_3 + b_3 - c_3)^3 \right]^{\frac{1}{3}} \right\} \right|.$$

$$\leq \left| \left\{ \frac{1}{3}\left[(a_1 - b_1)^3 + (b_1 - c_1)^3 + (a_2 - b_2)^3 + (b_2 - c_2)^3 + (a_3 - b_3)^3 + (b_3 - c_3)^3 \right]^{\frac{1}{3}} \right\} \right|$$

$$\leq \left| \left\{ \frac{1}{3} \big[(a_1 - b_1)^3 + (a_2 - b_2)^3 + (a_3 - b_3)^3 \big]^{\frac{1}{3}} \right\} \right|$$

$$+ \left| \left\{ \frac{1}{3} \big[(b_1 - c_1)^3 + (b_2 - c_2)^3 + (b_3 - c_3)^3 \big]^{\frac{1}{3}} \right\} \right|$$

$$\leq FS_{Tri}dis(\tilde{A}, \tilde{B}) + FS_{Tri}dis(\tilde{B}, \tilde{C}).$$

Thus $FS_{Tri}dis(\tilde{A}, \tilde{C}) \leq FS_{Tri}dis(\tilde{A}, \tilde{B}) + FS_{Tri}dis(\tilde{B}, \tilde{C}).$
Hence the Proof.

Example: 1
Let us consider the following example:

Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8), \tilde{B} = (0.3, 0.5, 0.7, 0.9) and \tilde{C} = (0.1, 0.2, 0.3, 0.4)$ are Trapezoidal Fuzzy Numbers.

(i)To Prove $FS_{Tri}dis(\tilde{A}, \tilde{B}) \geq 0.$

Solution:

$$FS_{Tri}dis(\tilde{A}, \tilde{B}) = \left| \left\{ \frac{1}{3} \big[(0.2 - 0.3)^3 + (0.4 - 0.5)^3 + (0.6 - 0.7)^3 \big]^{\frac{1}{3}} \right\} \right|$$

$$= \left| \frac{1}{3}(-0.1442) \right| = 0.04807 \geq 0.$$

Hence Proved.

(ii)To Prove $FS_{Tri}dis(\tilde{A}, \tilde{B}) = FS_{Tri}dis(\tilde{B}, \tilde{A})$

Solution:

$L.H.S$:

$$FS_{Tri}dis(\tilde{A}, \tilde{B}) = \left| \left\{ \frac{1}{3} \big[(0.2 - 0.3)^3 + (0.4 - 0.5)^3 + (0.6 - 0.7)^3 \big]^{\frac{1}{3}} \right\} \right|$$

$$= \left| \frac{1}{3}(-0.1442) \right| = 0.04807 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

$R.H.S$:

$$FS_{Tri}dis(\tilde{B}, \tilde{A}) = \left| \left\{ \frac{1}{3} \big[(0.3 - 0.2)^3 + (0.5 - 0.4)^3 + (0.7 - 0.6)^3 \big]^{\frac{1}{3}} \right\} \right|$$

$$= \left| \frac{1}{3}(0.1442) \right| = 0.04807 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2)$$

From (1) and (2),

L.H.S = R.H.S.

Thus $FS_{Tri}dis(\tilde{A}, \tilde{B}) = FS_{Tri}dis(\tilde{B}, \tilde{A}).$

Hence Proved.

(iii)To Prove $FS_{Tri}dis(\tilde{A},\tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}$.

Solution:

Let us consider $\tilde{A} = (0.2,0.4,0.6,0.8), \tilde{B} = (0.2,0.4,0.6,0.8)$ be two trapezoidal fuzzy numbers.

If $\tilde{A} = \tilde{B}$ then,

$$FS_{Tri}dis(\tilde{A},\tilde{B}) = \left| \left\{ \frac{1}{3}\left[(0.2-0.2)^3 + (0.4-0.4)^3 + (0.6-0.6)^3 \right]^{\frac{1}{3}} \right\} \right|$$

$$= 0.$$

Thus $\tilde{A} = \tilde{B} \Rightarrow FS_{Tri}dis(\tilde{A},\tilde{B}) = 0$.

Conversely, If $FS_{Tri}dis(\tilde{A},\tilde{B}) = 0$ then,

$$FS_{Tri}dis(\tilde{A},\tilde{B}) = 0$$

$$\Rightarrow \left| \left\{ \frac{1}{3}\left[(0.2-0.2)^3 + (0.4-0.4)^3 + (0.6-0.6)^3 \right]^{\frac{1}{3}} \right\} \right| = 0$$

$$\Rightarrow (0.2-0.2)^3 + (0.4-0.4)^3 + (0.6-0.6)^3 = 0$$

$$\Rightarrow (0.2-0.2)^3 = 0, (0.4-0.4)^3 = 0, (0.6-0.6)^3 = 0$$

$$\Rightarrow 0.2 = 0.2, 0.4 = 0.4, 0.6 = 0.6.$$

$$\Rightarrow \tilde{A} = \tilde{B}.$$

Thus $FS_{Tri}dis(\tilde{A},\tilde{B}) = 0 \Rightarrow \tilde{A} = \tilde{B}$.

Hence Proved.

(iv)To Prove $FS_{Tri}dis(\tilde{A},\tilde{C}) \leq FS_{Tri}dis(\tilde{A},\tilde{B}) + FS_{Tri}dis(\tilde{B},\tilde{C})$.

Solution:

L.H.S :

$$FS_{Tri}dis(\tilde{A}, \tilde{C}) = \left| \left\{ \frac{1}{3} \left[(0.2 - 0.1)^3 + (0.4 - 0.2)^3 + (0.6 - 0.3)^3 \right]^{1/3} \right\} \right|$$

$$= \frac{1}{3}(0.036)^{1/3} = \frac{0.3302}{3} = 0.11006 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

R.H.S :

$$FS_{Tri}dis(\tilde{A}, \tilde{B}) + FS_{Tri}dis(\tilde{B}, \tilde{C})$$

$$= \left| \left\{ \frac{1}{3} \left[(0.2 - 0.3)^3 + (0.4 - 0.5)^3 + (0.6 - 0.7)^3 \right]^{1/3} \right\} \right| +$$

$$\left| \left\{ \frac{1}{3} \left[(0.3 - 0.1)^3 + (0.5 - 0.2)^3 + (0.7 - 0.3)^3 \right]^{1/3} \right\} \right|$$

$$= 0.04806 + 0.1542$$

$$= 0.20226 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From equations (3) and (4)

L.H.S $\leq$ R.H.S.

Thus $FS_{Tri}dis(\tilde{A}, \tilde{C}) \leq FS_{Tri}dis(\tilde{A}, \tilde{B}) + FS_{Tri}dis(\tilde{B}, \tilde{C})$.
Hence Proved.

CONCLUSION
The main aim of this paper is to introduce new properties and the theorem based on Fuzzy Sub-Trident Distance. The advantage of this paper is simple and easy to apply and to solve Transportation Problems.

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